

Math 245 - Practice Quiz # 4

Directions: Show **ALL** of your work. Answers that are not supported by calculations, graphs/diagrams, and explanations will **not** be given full credit.

1. (4 total points - 1 point each) Please circle either T (true) or F (false) for each of the below statements. There is no penalty for guessing. You DO NOT have to show your work to receive full credit.

I) T F The chain rule for smooth functions $u(x, y)$, $x(t)$, and $y(t)$ says that $z(t) = u(x(t), y(t))$ has the derivative

$$\frac{dz}{dt} = u_{xx}x'(t) + 2u_{xy}x'(t)y'(t) + u_{yy}y'(t).$$

II) T F For any linear function $f(x, y)$, $f_x = f_y = 0$.

III) T F If $f(x, y)$ is continuous at $(a, b) \in \mathbb{R}^2$, it must also be differentiable at (a, b) .

IV) T F The vector $\mathbf{n} = (2, 4, -1)$ is normal to the surface $z = x^2 + y^2$ at $(1, 2, 5)$.

2. (6 points) Let $f(x, y) = \ln(x^2 + y)$. Find the rate of change of f at $(1, 1)$ in the direction of the vector $\mathbf{v} = -3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$.

3. (6 points) Compute $\partial/\partial x$ and $\partial/\partial y$ for both $f(x, y)$ and $g(x, y)$ below:

$$f(x, y) = 3x^2y^5 - \ln(xy^2) \quad \text{and} \quad g(x, y) = \tan^{-1}\left(\frac{x}{xy+1}\right) + \tan(e^{x-y}).$$

4. (4 points) Use the chain rule to compute u_s and u_t if

$$u(x, y) = e^{x^2-y^2} \quad \text{with} \quad x(s, t) = \frac{s}{t+1} \quad \text{and} \quad y(s, t) = \sec(st).$$