

Math 245 - Practice Problems for Quiz # 3

1. Please circle either T (true) or F (false) for each of the below statements. There is no penalty for guessing.

- I) T F The chain rule for smooth functions $u(x, y)$, $x(s, t)$, and $y(s, t)$ says that $z(s, t) = u(x(s, t), y(s, t))$ has the second partial derivative

$$\frac{\partial^2 z}{\partial t^2} = u_{xx}x_t^2 + 2u_{xy}x_t y_t + u_{yy}y_t^2.$$

- II) T F $\lim_{(x,y) \rightarrow (2,1)} (2x - 3y) = 1$.

- III) T F The domain of $\ln(x^2 + y^2)$ is $\mathbb{R}^2$.

- IV) T F The chain rule for smooth functions $u(x, y)$, $x(t)$, and $y(t)$ says that $z(t) = u(x(t), y(t))$ has the derivative

$$\frac{dz}{dt} = u_{xx}x'(t) + 2u_{xy}x'(t)y'(t) + u_{yy}y'(t).$$

- V) T F For any linear function $f(x, y)$, $f_x = f_y = 0$.

- VI) T F If $f(x, y)$ is continuous at $(a, b) \in \mathbb{R}^2$, then $\lim_{(x,y) \rightarrow (a,b)} f(x, y) = 0$.

2. Compute the below partial derivatives:

$$\frac{\partial}{\partial x} (x^2 y - \ln(x + y)) \quad \text{and} \quad \left(\sin^2 \left(\frac{x + y}{x - y} \right) \right)_y.$$

3. Find and graph in the x - y plane the domain of

$$f(x, y) = \frac{xy}{1 - x^2 - y^2}.$$

4. Determine whether or not the limit

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y - x^2 - y^2}{x^2 + y^2}$$

exists. If it does not, prove your conclusion. If it does, demonstrate why and find its value?

5. Compute $\partial/\partial x$ and $\partial/\partial y$ for both $f(x, y)$ and $g(x, y)$ below:

$$f(x, y) = 3x^2 y^5 - \ln(xy^2) \quad \text{and} \quad g(x, y) = \tan^{-1} \left(\frac{x}{xy + 1} \right) + \tan(e^{x-y}).$$

6. Use the chain rule to compute u_s and u_t if

$$u(x, y) = e^{x^2 - y^2} \quad \text{with} \quad x(s, t) = \frac{s}{t + 1} \quad \text{and} \quad y(s, t) = \sec(st).$$