

Math 245 Multivariable Calculus - Practice Problems for Exam # 2

1. Evaluate the following partial derivatives.

A)

$$\frac{\partial}{\partial x} (\sin(xy) + x^y - \ln(x+y))$$

$$\text{Answer: } y \cos(xy) + yx^{y-1} - \frac{1}{x+y}.$$

B)

$$\frac{\partial^2}{\partial y \partial x} (\sin(xy) + x^y - \ln(x+y))$$

$$\text{Answer: } \cos(xy) - xy \sin(xy) + \left(\frac{y \ln x + 1}{x} \right) x^y + \frac{1}{(x+y)^2}$$

2. Consider the function $f(x, y) = 2 \sin(2x - 3y)$. Find the

(a) Find ∇f .

$$\text{Answer: } \nabla f = (4 \cos(2x - 3y), -6 \cos(2x - 3y)).$$

(b) Find the rate of change of f at $(0, \pi)$ in the direction $\hat{\mathbf{i}} + \hat{\mathbf{j}}$.

$$\text{Answer: } \sqrt{2}.$$

(c) In which direction at the point $(0, \pi)$ is the rate of change of f zero. Give your answer as a unit vector.

$$\text{Answer: } \frac{3\hat{\mathbf{i}} + 2\hat{\mathbf{j}}}{\sqrt{13}}.$$

3. Find the equation of the tangent plane to the surface $z = x^2 e^{x-y}$ at $(2, 2, 4)$.

$$\text{Answer: } 8x - 4y - z = 4.$$

4. Consider

$$f(x, y) = x^4 + 2y^2 - 4xy.$$

A) Find all critical points of f .

$$\text{Answer: } (-1, -1), (0, 0), (1, 1).$$

- B) Use the second derivative test to analyze your answers to part (A), identifying all maxima, minima, or saddle points.

Answer: $(-1, -1)$ is a local minimum, $(0, 0)$ is a saddle point, and $(1, 1)$ is a local minimum.

5. Consider the function

$$f(x, y) = e^{xy} \sin(x - y).$$

Verify that $f_{xy} = f_{yx}$.

6. Determine whether or not the function

$$u(x, y) = \sin x \cosh y + \cos x \sinh y$$

is a solution of Laplace's Equation $\Delta u = u_{xx} + u_{yy} = 0$.

Answer: It does.

7. The radius of a right circular cone is increasing at a rate of 7 cm/sec while its height is decreasing at a rate of 20 cm/sec.* How fast is the volume changing when $r = 45$ cm and $h = 100$ cm? Is the volume increasing or decreasing?

Answer: $\frac{dV}{dt} = 7500 \pi \frac{\text{cm}^3}{\text{sec}} \simeq 23,561.9 \frac{\text{cm}^3}{\text{sec}}$.

8. Consider

$$f(x, y) = x^3 + 3xy^2 + 3y^2 - 15x + 2.$$

- A) Find all critical points of f .

Answer: $(-1, -2)$, $(-1, 2)$, $(\sqrt{5}, 0)$, $(-\sqrt{5}, 0)$.

- B) Use the second derivative test to analyze your answers to part (A), identifying all maxima, minima, or saddle points.

Answer: $(-1, \pm 2)$ are saddle points, $(\sqrt{5}, 0)$ is a minimum and $(-\sqrt{5}, 0)$ is a maximum.

*The volume V of a right circular cone of height $h > 0$ and radius $r > 0$ is

$$V = \frac{1}{3} \pi r^2 h.$$

9. Find three positive numbers whose sum is 100 and whose product is a maximum.

Answer: $x = y = z = 100/3$. The maximum product is $10^6/27$.

10. Find the rectangular box with the largest surface area in the first octant with three faces in the coordinate planes and one vertex on the plane $2x + 2y + z = 14$. Ensure that you fully justify your solution.

Answer: Maximum surface area of 56 with $x = 2$, $y = 2$, and $z = 6$.