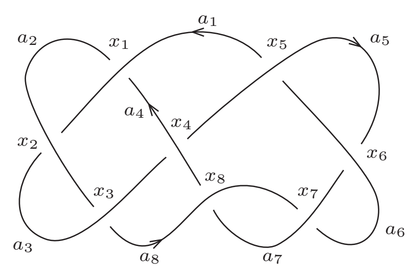
Additional Exercises for Chapter 2 of *Topology Now!*

1. Consider a Möbius band formed by gluing together the ends of a rectangular strip with one half-twist. Consider another Möbius band formed by gluing together the ends of a rectangular strip with three half-twists.
   a. Show that the two objects are homeomorphic.
   b. Show that the boundary curves of these two Möbius bands are different knots in $\mathbb{R}^3$.
   c. Conclude that the Möbius bands are not ambient isotopic in $\mathbb{R}^3$.

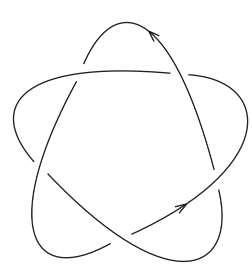
2. a. Look up the Alexander polynomials of the 21 knots of crossing number 8 and compute the determinants of these knots.
   b. Use the result stated in Exercise 8 in Section 2.5 to determine which of these knots are 3-colorable.
   c. Use the result stated in Exercise 8 in Section 2.5 to determine which of these knots are 5-colorable.

3. a. Use the labels and orientation on the knot below to compute its Alexander polynomial. Feel free to use a computer algebra system to compute the determinant of the reduced crossing/arc matrix.



   b. Identify this knot in a table of knots.
   c. For what primes p can this knot be colored with p colors? Find such colorings.

4. Use the skein relation for the Jones polynomial to compute this invariant of the oriented knot below. Use the fact that the Jones polynomial of the right-handed trefoil knot is $t + t^3 - t^4$ and the Jones polynomial of the right-handed Hopf link is $-t^{1/2} - t^{5/2}$.



5. Why is $t^6 - 3t^5 + 5t^4 - 9t^3 + 5t^2 - 3t + 1$ not the Alexander polynomial of any knot?
6. Look up the Jones polynomials of the 21 knots of crossing number 8. It is known that exactly five of these knots are achiral. Which ones are they?