Math 312 - Quiz #4 - Solution

1. (1 point each) Please circle either T (true) or F (false) for each of the below statements. **Answers are in BOLD.**

   A) T **F** Bonds are used by individuals to borrow money from the government or a company.
   B) **T** F If a bond defaults then the owner will not receive future payments from the bond issuer.
   C) **T** F The redemption value of a bond can be less than the face value.
   D) T **F** The dirty bond price in between coupon payments accounts for the value of the next coupon payment.

2. (4 points) A 12-year, \$20,000 par bond pays semiannual coupons at a nominal rate of 4.4% and redeems at \$21,000. If the bond is priced to yield an effective annual rate of 8%, find the price of the bond.

   I) \$11,655 II) \$14,511 III) \$14,704 IV) \$14,901 V) \$15,101

   Solution: For a bond that redeems at R, the price is given by

   $$P = R\nu^n + F \cdot r \cdot a_{\overline{n}|j}.$$

   For this problem, $n = 24$, $r = 0.044/2 = 0.022$, $R = 21{,}000$, $F = 20{,}000$, and $j = \sqrt{1.08} - 1 \simeq 0.0392305$. Hence,

   $$P = \frac{21{,}000}{1.08^{12}} + 20{,}000 \cdot 0.022 \cdot a_{\overline{24}|0.0392305} \simeq \boxed{\$15{,}101.20.}$$

   $\therefore$ **the correct answer is V.**

3. (4 points) A 30-year bond with a par value of 1000 and 12% coupons payable quarterly is selling for 850. Find $i^{(4)}$, the annual nominal yield rate, convertible quarterly.

   Solution: For the case of quarterly coupons, the coupon rate is $r = 0.12/4 = 0.03$ and the coupon amount is $F \cdot r = 1000 \cdot 0.03 = 30$. Further, there are $30 \cdot 4 = 120$ coupons over the course of 30 years. Using the BA-II+ calculator with $\boxed{\text{N}} = 120$, $\boxed{\text{PMT}} = 30$, $\boxed{\text{FV}} = 1000$, $\boxed{\text{PV}} = -850$, we get via $\boxed{\text{CPT}} + \boxed{\text{I/Y}}$ that $\boxed{\text{I/Y}} = 3.539178465$ or $j \simeq 0.03539$ as the quarterly yield rate. It follows that the nominal annual rate, convertible quarterly is

   $$\boxed{i^{(4)} \simeq 4(0.03539) \simeq 14.2\%.}$$

4. (4 points) A 10-year \$1,000 par bond with 6% semiannual coupons is purchased to yield 5.6% convertible semiannually. How much premium is amortized in the seventh coupon payment?

I) 1.33 II) 1.36 III) 1.39 IV) 1.42 V) 1.45

Solution: The amortized premium amount in payment t is given by

$$M_t = F \cdot (r - j) \cdot \nu^{n-t+1},$$

where $F = 1000$, $n = 20$, $t = 7$, $r = 0.06/2 = 0.03$, and $j = 0.056/2 = 0.028$. Hence,

$$M_7 = 1000 \cdot (0.03 - 0.028) \cdot \left(\frac{1}{1 + 0.028}\right)^{20-7+1} = \frac{1000 \cdot 0.002}{1.028^{14}} \simeq \boxed{1.35871.}$$

$\therefore$ **the correct answer is II.**

5. (4 points) Hannah has a 14-year 6% semi-annual coupon bond purchased to yield an interest rate of 4.5% convertible semiannually. The amount of premium amortized in the 4^{th} coupon is 14. If the bond is redeemable at par, find the book value of the bond immediately after the 6^{th} coupon.

I) 3,256 II) 3,676 III) 3,759 IV) 4,413 V) 4,711

Solution: F is not given. However, using $n = 28$, $j = 0.045/2 = 0.0225$, $r = 0.06/2 = 0.03$, and $t = 4$, we have

$$M_t = M_4 = 14 = F \cdot (0.03 - 0.0225) \cdot (1.0225)^{-(28-4+1)} \quad \Rightarrow \quad F = \frac{14 \cdot 1.0225^{25}}{0.0075} \simeq 3255.74.$$

It follows that the book value immediately after the 6^{th} coupon is the present value, at time $t = 6$, of the remaining coupons and redemption amount:

$$P_6 = F \cdot \nu^{22} + F \cdot r \cdot a_{\overline{22}|j} = \frac{3255.74}{1.0225^{22}} + (3255.74) \cdot (0.03) \cdot a_{\overline{22}|0.0225} \simeq \boxed{3{,}675.81.}$$

$\therefore$ **the correct answer is II.**