Math 312 - Mathematical Theory of Interest - Quiz #2 - Solution

1. (1 point each) Please circle either T (true) or F (false) for each of the below statements. **Answers are in BOLD.**

   A) **T** F $(Is)_{\overline{\infty}|0.1}$ is undefined.

   B) **T** F $a_{\overline{n}|i} = \ddot{a}_{\overline{n}|i} \cdot \nu$.

   C) T **F** $\ddot{s}_{\overline{360}|0.004}$ is the accumulated value of a sequence of 360 payments of 1, at the time of the final payment, with a per-period interest rate 0.4%.

   D) T **F** $(Ia)_{\overline{\infty}|0.2} = 5$.

2. (6 points) Eric dreams of owning a *Porsche Cayenne*. Three years from today, the projected cost of the 2028 model is \$121,007. To save for the 20% down payment he will need in 3 years, Eric plans to deposit X into an investment account at the end of every quarter, starting one quarter from today. If Eric's account earns continuous interest at an annual rate of 12%, find X.

   A) 1650

   B) 1655

   C) 1700

   D) 1705

   E) 1755

   <u>*Solution:*</u> 20% of \$121,007 is \$24,201.40 and there are 12 quarters in 3 years. Therefore X must satisfy

$$\$24{,}201.20 = X \cdot s_{\overline{12}|j},$$

where j is the quarterly interest rate given by $j = e^{0.12/4} - 1 \simeq 0.0304545$. Hence,

$$X = \frac{\$24{,}201.20}{s_{\overline{12}|0.0304545}} = \frac{24{,}201.20}{\frac{(1.0304545)^{12}-1}{0.0304545}} \simeq \boxed{\$1{,}700.87.}$$

$\therefore$ **the correct answer is C.**

3. (5 points) An annuity pays 100 at the end of each of the next 5 years and 300 at the end of each of the five following years. If $i^{(12)} = 6\%$, find the present value of the annuity.

<u>*Solution:*</u> The present value of the first five 100 payments is

$$100 \cdot a_{\overline{5}|j} \quad \text{where} \quad j = \left(1 + \frac{i^{(12)}}{12}\right)^{12} - 1 = 1.005^{12} - 1 \simeq 0.0616778.$$

Moreover, the 300 portion of the annuity, **at time** $t = 5$, is $300 \cdot a_{\overline{5}|j}$. Discounting this portion of the amount back to time $t = 0$ (5 total years) and adding to the above yields at total present value of

$$PV = 100 \cdot a_{\overline{5}|0.0616778} + \nu^5 \cdot 300 \cdot a_{\overline{5}|0.0616778} \simeq \boxed{1{,}351.94.}$$

4. (5 points) A perpetuity-immediate pays 100 per year. Immediately after the fifth payment, the perpetuity is exchanged for a 25 year annuity-immediate that will pay X at the end of the first year. Each subsequent annual payment will be 8% greater than the preceding payment. The annual effective rate is 8%.

Calculate X.

A) 54
B) 64
C) 74
D) 84
E) 94

<u>*Solution:*</u> The present value of the annuity at *ANY TIME* right after a payment is

$$PV = 100 \cdot a_{\overline{\infty}|0.08} = \frac{100}{0.08} = 1250.$$

The 25 year annuity payment, as described, has value at the time of exchange given by

$$\begin{aligned} 1250 &= X\nu + X \cdot 1.08 \cdot \nu^2 + X \cdot 1.08^2 \cdot \nu^3 + X \cdot 1.08^3 \cdot \nu^4 + \cdots + X \cdot 1.08^{24} \cdot \nu^{25}, \\ &= X \cdot \sum_{j=1}^{25} 1.08^{j-1} \cdot \nu^j = X \cdot \underbrace{\sum_{j=1}^{25} \frac{1.08^{j-1}}{1.08^j}}_{\nu = 1.08^{-1}} = X \cdot \sum_{j=1}^{25} \frac{1}{1.08} = \frac{25X}{1.08} \end{aligned}$$

It follows that

$$X = \frac{1250 \cdot 1.08}{25} \simeq \boxed{54.}$$

$\therefore$ **the correct answer is A.**