Math 312 - Quiz # 1 - Solution

1. (1 point each) Please circle either T (true) or F (false) for each of the below statements. **Answers are in BOLD.**

   A) T **F** If the simple interest rate is 20%, then the value of 1000 in 10 years 1200.

   B) T **F** If $i^{(3)} = 12\%$, then the equivalent 1-month rate is 1%.

   C) **T** F If the annual effective interest rate is 1/10, then the annual effective discount rate is 1/11.

   D) **T** F If the force of interest is constant $\delta = 0.05$, then the present value of 1, 20 years in the future, is e^{-1}.

2. (6 points) You perform the following transactions:

   - Deposit 3500 into an account that earns 6% nominal annual interest, compounded monthly;

   - After two years you deposit 500 more into the account;

   - After four more years you move all of the money into a new account that earns 4% nominal annual discount rate, compounded quarterly.

   What is the value of your account 9 years after the initial 3500 deposit?

   Solution: The value of the account after 2 years with the 500 deposit is

   $$A(2) = 3500 \left(1 + \frac{0.06}{12} \right)^{12 \cdot 2} + 500 = 3945.06 + 500 = 4445.06.$$

   The entire account continues to grow at $i^{(12)} = 6\%$ for four more years, resulting in a final value of

   $$A(6) = 4445.06(1.005)^{4 \cdot 12} \simeq 5647.40.$$

   Finally, three more years of growth at a nominal annual discount rate of 4%, compounded quarterly, results in an account value of

   $$A(9) = \frac{5647.40}{\left(1 - \frac{d^{(4)}}{4} \right)^{3 \cdot 4}} = \frac{5647.40}{\left(1 - \frac{0.04}{4} \right)^{12}} \simeq \boxed{6371.27.}$$

3. (5 points) The value of an account at time t is $A(t) = 1000 + 50t$. At what time t is the force of interest 2.5%?

Solution: Recall that the force of interest $\delta(t)$ satisfies $\delta(t) = A'/A$. Hence,

$$\delta(t) = \frac{(1000 + 50t)'}{(1000 + 50t)} = \frac{50}{1000 + 50t} = \frac{1}{20 + t}.$$

It follows that

$$\delta(t) = \frac{1}{20 + t} = 2.5\% = 0.025 = \frac{1}{40} \quad \Rightarrow \quad 20 + t = 40 \quad \Rightarrow \quad \boxed{t = 20.}$$

4. (5 points) Hannah and Abby each open up new bank accounts at time 0, respectively depositing 100 and 50 into their accounts. Each account earns the same annual effective interest rate. The amount of interest X earned in Hannah's account during the 11th year is equal to the amount of interest earned in Abby's account during the 17th year. Find X.

I) 28.0

II) 31.3

III) 34.6

IV) 36.7

V) 38.9

Solution: Let i be the annual effective interest rate. The amount of interest X earned in Hannah's account during the 11th year is

$$X = i \cdot 100 \cdot (1 + i)^{10}$$

while the amount of interest X earned in Abby's account during year 17 is

$$X = i \cdot 50 \cdot (1 + i)^{16}.$$

It follows that

$$50 \cdot \cancel{i} \cdot (1 + i)^{\cancel{16}6} = 100 \cdot \cancel{i} \cdot \cancel{(1 + i)^{10}} \quad \Rightarrow \quad (1 + i)^6 = 2 \quad \Rightarrow \quad i = 2^{\frac{1}{6}} - 1 \simeq 0.122462.$$

The amount of interest X is therefore

$$X = i \cdot 100 \cdot (1 + i)^{10} = (2^{1/6} - 1) \cdot 100 \cdot \left(2^{\frac{1}{6}}\right)^{10} \simeq \boxed{38.87.}$$

∴ **The correct answer is V.**