Math 312 - Practice Problems for Quiz # 4 - Solution

1. A \$5000 10-year 10% bond is priced to yield 8% convertible semiannually and redeems at par.

   A) Without computing the price, explain how you can tell whether this bond is bought at a discount or at a premium.

   Solution: A bond that redeems at face value is bought at a discount if the coupon rate is smaller than the yield rate. On the other hand, if the coupon rate is higher than the yield rate the bond is bought at a premium. This follows from the formula (see bottom of page 230 in the text):

   $$P = F + F(r - j)\, a_{\overline{n}|j}.$$

   B) Find the price of the bond at $t = 0$ to confirm your answer to part (A).

   Solution: The price is

   $$P = F + F(r - j)a_{\overline{n}|j} = 5000 + 5000(.05 - .04)a_{\overline{40}|.04} \simeq \boxed{\$5679.52.}$$

2. A 20-year \$1000 bond that redeems at 1100 pays 8% semiannual coupons and is priced to yield at 4.6% convertible semiannually. If upon receipt the coupons are immediately reinvested into a fund that pays 7% interest compounded semiannually, find the effective annual interest rate of this investment.

   Solution: The price of the bond is

   $$P = \frac{1100}{(1 + .023)^{40}} + (1000 \cdot 0.04)a_{\overline{40}|.023} \simeq \$1481.76.$$

   If each \$40 coupon is invested upon receipt into a fund that earns $i^{(2)} = 7\%$, then the accumulated value in that account after 20 years is

   $$40\, s_{\overline{40}|.035} \simeq \$3382.01.$$

   Since at maturity the investor also receives \$1100, it follows that the effective annual yield of the investment j must satisfy

   $$1481.76\,(1 + j)^{20} = 3382.01 + 1100 = 4482.01 \quad \Rightarrow \quad \boxed{j = \sqrt[20]{\frac{4482.01}{1481.76}} \simeq 0.057 = 5.7\%.}$$

3. A 10-year bond pays semiannual coupons at a nominal rate of 5%. These coupons are invested at the end of every 6-month period into an account earning an effective annual rate of 7.5%. At the end of 10 years the value account is \$6,123.21. Find the face value of the original bond.

<u>*Solution:*</u> Let F be the face value of the 10-year bond. Its coupons are thus $F \cdot r = 0.025F$. The equivalent semi-annual rate for 7.5% is

$$k = \sqrt{1.075} - 1 \simeq 0.036822.$$

It follows that the accumulated value of the coupons is

$$0.025F \cdot s_{\overline{20}|0.036822} = 6{,}123.21 \quad \Rightarrow \quad F = \frac{\$6{,}123.21}{0.025 \cdot s_{\overline{20}|0.036822}} \simeq \boxed{\$8{,}500.00.}$$

4. An n-year \$5,000 par bond redeems at \$5,500 and pays 9% annual coupons and is priced at \$6,333.78 to redeem at an annual effective rate of 6.5%. Find the price of a bond with the same maturity n and yield except that it redeems at a par value of \$2,500 and pays annual coupons at a rate of 7%.

<u>*Solution:*</u> The n-year bond satisfies

$$\$6{,}333.78 = \$5{,}500\,\nu^n + (\$5{,}000 \cdot 0.09) \cdot a_{\overline{n}|0.065} = \$5{,}500\,\nu^n + \frac{\$5{,}000 \cdot 0.09}{0.065} \cdot (1 - \nu^n)\,.$$

Solving for ν^n yields

$$\nu^n \simeq 0.4141 \quad \Rightarrow \quad (1.065)^{-n} = 0.4141 \quad \Rightarrow \quad n = -\frac{\ln(0.4141)}{\ln(1.065)} \simeq 14.$$

It follows that the \$2,500 bond with 7% coupons has the price

$$P = \$2{,}500 + \$2{,}500 \cdot (.07 - .065) \cdot a_{\overline{14}|0.065} \simeq \boxed{\$2{,}612.67.}$$

5. A 10-year \$1,000 par bond with semiannual 8% coupons and redemption value \$1200 is purchased for \$1,855.41. Find the price of a 12-year \$1,300 par bond with 6% semiannual coupons that redeems at \$1075. Both bonds priced at the same semiannual yield rate.

I) \$1,722.47
II) \$1,844.89
III) \$1,855.41
IV) \$1,927.47
V) \$2,046.10

Solution: Note that the yield is not given. From the first bond we have

$$\$1{,}855.41 = \frac{\$1{,}200}{(1+j)^{20}} + \$1{,}000 \cdot (.04) \cdot a_{\overline{20}|j}.$$

Using the BA-II+ calculator with $\boxed{\text{N}} = 20$, $\boxed{\text{PMT}} = 40$, $\boxed{\text{FV}} = 1200$, $\boxed{\text{PV}} = -1855.41$, we get via $\boxed{\text{CPT}} + \boxed{\text{I/Y}}$ that $\boxed{\text{I/Y}} = 0.46667$ or $j \simeq 0.0046667$ as the semiannual yield rate. It follows that the price of the 12-year bond is

$$P = \frac{\$1{,}075}{1.0046667^{24}} + (\$1{,}300) \cdot (0.03) \cdot a_{\overline{24}|0.0046667} \simeq \boxed{\$1{,}844.89.}$$

The correct answer is II.