

### Math 312 - Practice Quiz # 3 - Solution

1. (1 point each) Please circle either T (true) or F (false) for each of the below statements.  
**Answers are in BOLD.**

- A) **T** **F**  $30000/a_{\overline{60}|0.06}$  represents the monthly loan payment amount on a \$30,000 car with a 5-year loan and level monthly payments at a nominal annual rate of 6%, compounded monthly.
- B) **T** **F** In an amortized loan, 100% of the interest owed on the outstanding balance must be repaid each period.
- C) **T** **F** The retrospective method for finding the outstanding loan balance at time  $t$  computes the difference between the future value of the original amount borrowed and the future value of all payments made on the loan.
- D) **T** **F** The interest rate of an amortized loan must be constant over the life of the loan.

2. (8 total points) You borrow 300,000 € with a 30-year mortgage at a nominal interest rate of  $i^{(12)} = 6\%$ .

- A) (4 points) What is the monthly level payment  $K$  for this loan?

Solution:  $K$  solves

$$300000 = K \cdot a_{\overline{360}|0.005} \Rightarrow \boxed{K \simeq 1798.65 \text{ €}}$$

- B) (4 points) You decide to refinance your loan at the end of the 10<sup>th</sup> year for 15 years at a nominal annual interest rate of 4.5% convertible monthly. What is the new monthly payment?

Solution: 20 year remain on the original loan so the outstanding balance is

$$OB = 1798.651575 \cdot a_{\overline{240}|0.005} \simeq 251057.1748 \text{ €}.$$

Refinancing over 15 years at  $i^{(12)} = 4.5\%$  results in the new payment  $K_{\text{new}}$  which solves

$$K_{\text{new}} a_{\overline{15(12)}|0.045/12} = 251057.1748 \Rightarrow \boxed{K_{\text{new}} \simeq 1920.57 \text{ €}}$$

3. (4 points) A company has a loan of 100,000 to be repaid with 30 annual end-of-year level payments. The principal and the interest in the 21<sup>st</sup> payment are the same. Find the principal repaid in the 10<sup>th</sup> payment.

A) 1862      B) 1871      C) 1884      D) 1901      E) 1913

Solution: Level payment for this loan are

$$K = \frac{100000}{a_{\overline{30}|j}}$$

where  $j$ , the effective annual rate, is not given. However, from Table 3.6 in the book, we also know that for a loan consisting of  $n$  level payments  $K$ , the interest repaid and principle repaid on the  $t^{\text{th}}$  payment are respectively

$$I_t = K(1 - \nu^{n-t+1}) \quad \text{and} \quad PR_t = K\nu^{n-t+1}.$$

It follows for our case of  $n = 30$  and  $t = 21$  that

$$I_{21} = K(1 - \nu^{30-21+1}) = K(1 - \nu^{10}) = PR_{21} = K\nu^{30-21+1} = K\nu^{10}.$$

Thus

$$2\nu^{10} = 1 \quad \Rightarrow \quad \nu = \frac{1}{1+i} = \sqrt[10]{\frac{1}{2}} \quad \Rightarrow \quad i = \sqrt[10]{2} - 1 \simeq 0.07177 = 7.177\%.$$

It follows that the level payment is

$$K = \frac{100000}{a_{\overline{30}|0.07177}} \simeq 8202.6814.$$

We conclude that principle repaid in the 10<sup>th</sup> payment is

$$PR_{21} = K\nu^{21} = \frac{8202.6814}{1.07177^{21}} \simeq \boxed{1913.34}.$$

**The correct answer is E.**

4. (4 points) Consider a 30-year home mortgage at a 6.6% nominal annual interest rate compounded monthly, with level end-of-month payments of 766.39. What is the first period in which the principle repaid is over 500?

- I) 269
- II) 274
- III) 278
- IV) 281
- V) 284

Solution: Recall that for a loan with  $n$  level payments of amount  $K$ , the principle repaid in period  $t$  is given by  $K\nu^{n-t+1}$ . Therefore we want to solve

$$766.39\nu^{360-t+1} \geq 500.$$

It follows that

$$\nu^{361-t} \geq \frac{500}{766.39} \Rightarrow 361 - t \leq \frac{\ln\left(\frac{500}{766.39}\right)}{\ln \nu}$$

since  $\ln \nu < 0$  and  $\ln(x)$  is monotonically increasing. Therefore

$$t \geq 361 - \frac{\ln\left(\frac{500}{766.39}\right)}{\ln \nu} \simeq 283.135.$$

Hence, the first time the amount of principle exceeds 500 is payment 284. **The correct answer is V.**