Math 312 - Practice Quiz # 2 - Solution

1. (1 point each) Please circle either T (true) or F (false) for each of the below statements. **Answers are in BOLD.**

A)	T	**F**	$20\ddot{a}_{\overline{\infty}|.01}$ represents the value of a perpetuity due at the time one period before the 1^{st} payment.
B)	**T**	F	$50\ddot{s}_{\overline{12}|0.01}$ represents the accumulated value of 12 payments of 50, at an interest rate of 1% per period, one period after the final payment.
C)	T	**F**	For the same interest rate i per period, $\ddot{s}_{\overline{n}|i}$ and $a_{\overline{n}|i}$ cannot be related mathematically.
D)	**T**	F	The expression $20s_{\overline{40}|.05}/\nu$ represents the accumulated value of an annuity, one period after the time of the final payment of 20, where 40 payments have been made and an interest rate of 5% has been applied per period.

2. (6 points) Jenn's goal for saving for the future is \$500,000. She can invest in an account that has a nominal interest rate of 9% compounded monthly.

A) (4 points) How much should she invest each month if she is to reach her investment goal at the end of 30 years on the date of the last payment?

Solution: Since $.09/12 = 0.0075$, the monthly payment K should satisfy the equation

$$K \cdot s_{\overline{360}|.0075} = 500000 \quad \Rightarrow \quad \boxed{K \simeq \$\,273.11.}$$

B) (2 points) How much does the amount she should invest each month *increase* if she waits 10 years to start saving?

Solution: By waiting for 10 years, the number of investment opportunities for Jenn reduces to 240. Therefore the new payment K_{new} solves

$$K_{\text{new}} \cdot s_{\overline{240}|.0075} = 500000 \quad \Rightarrow \quad K_{\text{new}} \simeq \$\,748.63.$$

The **increase** is therefore $\boxed{\$748.63 - \$273.11 \simeq \$475.52.}$

3. (5 points) A 10-year annuity-immediate pays 50 quarterly for the first 5 years and 100 monthly for the last 5 years. The annuity earns at a nominal annual rate of 6% compounded quarterly. What is the present value of this annuity?

Solution: Treat this as the sum of a quarterly annuity immediate for the first 5 years with quarterly payments of 50 at $.06/4 = .015$ per quarter plus a deferred annuity immediate with payments of 100 monthly for the last 5 years at j per month, where j solves

$$(1+j)^{12} = (1.015)^4 \quad \Rightarrow \quad j = (1.015)^{1/3} - 1 \simeq 0.004975.$$

Therefore

$$PV = \underbrace{50a_{\overline{20}|.015}}_{\text{first 5 years}} + \underbrace{100 \cdot (1.015)^{-20} \cdot a_{\overline{60}|.004975}}_{\text{last 5 years}} \simeq \boxed{4701.74.}$$

4. (5 points) To accumulate 8000 at the end of $3n$ years, deposits of 98 are made at the end of each of the first n years and 196 at the end of each of the next $2n$ years. The annual effective interest rate is i. You are given $(1+i)^n = 2$. Find i.

A) 11.25%
B) 11.75%
C) 12.25%
D) 12.75%
E) 13.25%

Solution: The accumulated value equation is

$$98 \cdot s_{\overline{n}|i}(1+i)^{2n} + 196 s_{\overline{2n}|i} = 98(1+i)^{2n} \cdot \frac{(1+i)^n - 1}{i} + 196 \cdot \frac{(1+i)^{2n} - 1}{i} = 8000.$$

Since $(1+i)^n = 2$ it follows that

$$98 \cdot 2^2 \cdot \frac{2-1}{i} + 196 \frac{2^2 - 1}{i} = \frac{392}{i} + \frac{588}{i} = 8000 \quad \Rightarrow \quad \boxed{i = 0.1225 = 12.25\%.}$$

The correct answer is C.