Math 312 - I - Practice Quiz # 1 - Solution

1. (1 point each) Please circle either T (true) or F (false) for each of the below statements.

   A) **T** F For an annual effective interest rate i, the annual discount factor ν represents the present value of 1, one year from now.

   B) T **F** If $i^{(12)} = 24\%$, then the value of 100 twelve years in the future is $100(1.02)^{24}$.

   C) **T** F A nominal annual interest rate of 6%, convertible quarterly is the same as an effective quarterly rate of 1.5%.

   D) **T** F If $A(t) = e^{t^2-t}$, then the force of interest is $\delta(t) = 2t - 1$.

2. (8 total points) At time $t = 0$ you and your friend both invest 1000€. Your account earns simple interest at a rate of 10%.

   A) (4 points) If your friend's account earns a nominal annual rate of 9%, compounded monthly, find the accumulated value in *both* accounts after 18 months.

   Solution: For $t = 1.5$, your account has value

   $$A = 1000(1 + 0.1t) = 1000(1.15) = \boxed{1150€.}$$

   On the other hand, your friend's account has value

   $$A = 1000\left(1 + \frac{0.09}{12}\right)^{18} = 1000\,(1.0075)^{18} = \boxed{1143.96€.}$$

   B) (4 points) If your account still earns simple interest at a rate of 10%, what is the continuously compounded annual interest rate r earned by your friend's account if the two accounts have equal value after 18 months?

   Solution: To have equal value we require that r solve

   $$1150 = 1000e^{1.5r} \quad \Rightarrow \quad r = \frac{2}{3}\ln\left(\frac{1150}{1000}\right) \simeq \boxed{9.317\%.}$$

3. (4 points) Darren deposits 1000 into his account on 01/01/2024. Sijia deposits 500 into a different account on 01/01/2025, and another 600 into her same account on 01/01/2026. On 01/01/2028 both Darren and Sijia have the same amount in their accounts. The accounts earn the same annual effective interest j. What is j? Assume that $j > -1$.

   I) 6.2%
   II) 6.4%
   III) 6.6%
   IV) 6.8%
   V) 7.0%

Solution: The value of Darren's account on 1/1/2028 is

$$A_D = 1000(1+j)^4.$$

On the other hand, the value of Sijia's account is

$$A_S = 500(1+j)^3 + 600(1+j)^2.$$

Setting $A_S = A_D$ yields

$$1000(1+j)^4 = 500(1+j)^3 + 600(1+j)^2.$$

$j > -1$ implies that

$$1000(1+j)^2 - 500(1+j) - 600 = 0 \quad \Rightarrow \quad 1+j = \frac{500 \pm \sqrt{500^2 - 4\cdot 1000 \cdot (-600)}}{2 \cdot 1000}$$

or

$$1+j = 1.06394 \quad \text{or} \quad 1+j = -0.56394.$$

Therefore, since $j > -1$, we have

$$\boxed{j \simeq 1.06394 - 1 = 6.394\%.}$$

Hence, the correct answer is **II**.

4. (4 points) The force of interest in an account is $\delta_t = .002t + .05$. If $A(4) = 1000$, find $A(10)$.

Solution: We have from $A'/A = \delta_t$ that

$$A(10) = A(4)e^{\int_4^{10} \delta_t \, dt} = 1000e^{\int_4^{10} 0.002t + 0.05 \, dt} = 1000e^{0.001t^2 + 0.05t\big|_{t=4}^{t=10}}$$

or

$$\begin{aligned} A(10) &= 1000e^{0.001\cdot 100 + 0.05\cdot 10 - (0.001\cdot 16 + 0.05\cdot 4)}, \\ &= 1000e^{0.1+0.5-0.016-0.2} = 1000e^{0.384} \simeq \boxed{1468.145.} \end{aligned}$$