Math 312 - Exam #2 Practice Problems - Solution

1. A 20-year \$10,000 par bond with 8% annual coupons, payable semiannually, redeems at \$11,000 and is priced at P to yield at an annual effective rate of 7.35%. Find P.

   Solution: The equivalent semiannual rate is $j = \sqrt{1.0735} - 1 \simeq 0.03609845$ and coupons in the amount of $(0.04) \cdot \$10{,}000 = \$400$. The price P is therefore

$$P = \underbrace{\$400 \cdot a_{\overline{40}|0.03609845}}_{\text{coupons}} + \underbrace{\frac{\$11{,}000}{1.0735^{20}}}_{\text{redemption}} \simeq \boxed{\$11{,}061.25.}$$

2. You take out a 15-year, \$300,000 mortgage at a 5.4% nominal annual interest rate, convertible monthly. You make level monthly amortization payments for five years, and then refinance with a new 30-year mortgage at a 3.6% nominal annual rate, convertible monthly. Both mortgages require level amortization payments at the end of each month.

   Find R, the size of each monthly payment under the 30-year refinanced mortgage.

   Solution: The initial monthly payment with $j = i^{(12)}/12 = 0.054/12 = 0.0045$ is

$$C^{i} = \frac{\$300{,}000}{a_{\overline{15 \cdot 12}|0.0045}} \simeq \$2{,}435.3598.$$

   After 5 years, the outstanding balance is

$$B_5 = C^{i}\, a_{\overline{10 \cdot 12}|0.0045} \simeq 225{,}430.648.$$

   For a new 30-year mortgage with $j = i^{(12)}/12 = 0.036/12 = 0.003$ on the outstanding balance, the payment is

$$K_f = \frac{\$225{,}430.648}{a_{\overline{30 \cdot 12}|0.003}} \simeq \boxed{\$1{,}024.91.}$$

3. To invest for a future purchase of \$10,000, you purchase a 10-year par-redeemable bond with face value F that pays semiannual coupons at a nominal annual rate of 7% and is priced to yield 5.75%. Upon receipt you reinvest the coupons at 5.2% convertible semi-annually with the goal that the F plus the accumulated value of the coupons is worth \$10,000 in 10 years, on the day of the last coupon payment. What is the annual effective yield for this overall investment?

   I) 5.65%
   II) 5.69%
   III) 5.73%
   IV) 5.77%
   V) 5.81%

   Solution: The 10-year bond with semiannual coupons pays 20 coupons at 3.5% of face per coupon, or $0.035 \cdot F$. These are invested at $j = .052/2 = 0.026$ every six months so that the accumulated value of the deposited coupons is

$$\mathrm{FV}_{coupons} = (0.035) \cdot F \cdot s_{\overline{20}|0.026} \simeq 0.90311782\, F.$$

   At the time of the final coupons payments, the total amount accumulated is

$$0.90311782\, F + F = \$10{,}000 \quad \Rightarrow \quad F = \frac{\$10{,}000}{1.90311782} \simeq \$5{,}254.5354.$$

   Since the bond redeems at par, it follows that the price of the bond at a yield of 5.75% is

$$P = \$5{,}254.5354 + \$5{,}254.5354 \cdot (0.035 - .02875) \cdot a_{\overline{20}|0.02875} \simeq \$5748.81896.$$

   The annual effective yield r thus satisfies

$$P \cdot (1+r)^{10} = \$10{,}000 \quad \Rightarrow \quad r = \sqrt[10]{\frac{\$10{,}000}{\$5{,}748.81896}} - 1 \simeq 0.05692 \simeq \boxed{5.69\%.}$$

   $\therefore$ **the correct answer is II.**

4. A 15-year \$1000 bond that redeems at \$1100 and pays 3.5% annual coupons is priced at \$1029.28. Find the book value of the bond immediately after the 10^{th} coupon payment.

   Solution: The semiannual yield is not given but can be found using the BA-II+ with $\boxed{\text{2ND}}$ + $\boxed{\text{CLR TVM}}$, followed by

$$\boxed{\text{PV}} = -1029.28, \quad \boxed{\text{PMT}} = 35, \quad \boxed{\text{FV}} = 1100, \quad \boxed{\text{N}} = 15,$$

and then ending with $\boxed{\text{CPT}} + \boxed{\text{I/Y}} \simeq 0.0375 = 3.75\%$. Therefore, immediately after the 10th coupon payment the book value is

$$B_5 = 35 \cdot a_{\overline{5}|0.0375} + \frac{1100}{1.0375^5} \simeq 1071.9796 \simeq \boxed{\$1{,}071.98.}$$

5. To fix the roof on your business, you borrow \$20,000 for 7 years from your local credit union. You agree to make interest only payments to the credit union at the end of every quarter plus fully repay the principal at the end of 7 years. To save for principal repayment of the principal you deposit X into a separate account at the end of every quarter for the next 4 years, followed by end-of-quarter deposits of $1.5X$ for the remaining 3 years.

   If this separate account earns $i^{(12)} = 5.4\%$, find X to the nearest dollar.

   I) \$497
   II) \$503
   III) \$509
   IV) \$515
   V) \$521

   <u>*Solution:*</u> The equivalent quarterly rate j satisfies

$$(1+j)^4 = \left(1 + \frac{0.054}{12}\right)^{12} \quad \Rightarrow \quad j = (1 + .054/12)^3 - 1 \simeq 0.013561.$$

   The accumulated value of the first 16 payments of X after 7 years is

$$X \cdot s_{\overline{16}|0.013561} \cdot (1.013561)^{12}$$

   and the accumulate value of the final 12 payments of $1.5X$ after 7 years is

$$1.5 \cdot X \cdot s_{\overline{12}|0.013561}.$$

   Adding together and setting it equal to the principal payment of \$20,000 yields

$$X \cdot s_{\overline{16}|0.013561} \cdot (1.013561)^{12} + 1.5 \cdot X \cdot s_{\overline{12}|0.013561} = \$20{,}000$$

or

$$X = \frac{\$20{,}000}{s_{\overline{16}|0.013561} \cdot (1.013561)^{12} + 1.5 \cdot s_{\overline{12}|0.013561}} \simeq \boxed{\$496.88.}$$

$\therefore$ **the correct answer is I.**

6. A 10-year \$1000 par bond redeems at par and pays semiannual coupons at a nominal rate of 8%. The premium amortized in the 6^{th} payment is \$11.47 and the premium amortized in the 12^{th} payment is \$13.20. Find the nominal annual yield on the bond, convertible semi-annually.

   I) 2.29%
   II) 2.37%
   III) 4.42%
   IV) 4.58%
   V) 4.74%

<u>*Solution:*</u> Let $\nu = (1+j)^{-1}$ where j is the 6-month yield rate and note that there are $n = 20$ coupon payments. It follows that the premium amortized in the 6^{th} and 12^{th} coupon payments are respectively

$$M_6 = F \cdot (r-j) \cdot \nu^{n-t+1}\Big|_{t=6,\, n=20} = \$1000 \cdot (0.04-j) \cdot \nu^{15} = \$11.47$$

and

$$M_{12} = F \cdot (r-j) \cdot \nu^{n-t+1}\Big|_{t=12,\, n=20} = \$1000 \cdot (0.04-j) \cdot \nu^{9} = \$13.20$$

Therefore

$$\frac{\cancel{\$1000 \cdot (0.04-j)} \cdot \nu^{15}}{\cancel{\$1000 \cdot (0.04-j)} \cdot \nu^{9}} = \frac{\$11.47}{\$13.20} = \nu^6.$$

Thus,

$$\nu = \frac{1}{1+j} = \sqrt[6]{\frac{11.47}{13.20}} \quad \Rightarrow \quad j = \sqrt[6]{\frac{13.20}{11.47}} - 1 \simeq 0.0473798 \quad \Rightarrow \quad \boxed{i^{(2)} = 2j = 4.74\%.}$$

$\therefore$ **the correct answer is V.**

7. An investor is asked to invest \$1000 and is promised in return a payment of \$380 in one year, \$256 in two years, and then \$540 in three years. Find her IRR.

   Solution: The TVM equation is

$$1000 = 380\nu + 256\nu^2 + 540\nu^3.$$

   Using the BAII and the CF ("cash flow") feature, which can find the IRR for an irregular cash flow, as follows:

   - Press $\boxed{\text{CF}}$ + $\boxed{\text{2ND}}$ + $\boxed{\text{CE|C}}$, clearing old work.
   - Press $\boxed{\text{CF}}$ + "CFo =" -1000 + $\boxed{\text{ENTER}}$.
   - Press $\boxed{\downarrow}$ + "C01 =" 380 + $\boxed{\text{ENTER}}$.
   - Press $\boxed{\downarrow}$ twice + "C02 =" 256 + $\boxed{\text{ENTER}}$.
   - Press $\boxed{\downarrow}$ twice + "C03 =" 540 + $\boxed{\text{ENTER}}$.
   - Press $\boxed{\text{IRR}}$ + $\boxed{\text{CPT}}$. This will give you $\boxed{8\%}$ as the answer.

8. You are given the following information about the activity in two different investment accounts:

   - Account K:

Date	**Fund Value Before Deposit/Withdrawal**	**Deposit**	**Withdrawal**
January 1, 2019	100.0		
July 1, 2019	125.0		X
October 1, 2019	110.0	$2X$	
December 31, 2019	125.0		

   - Account L:

Date	**Fund Value Before Deposit/Withdrawal**	**Deposit**	**Withdrawal**
January 1, 2019	100.0		
July 1, 2019	125.0		X
December 31, 2019	105.8		

During 2019, the dollar-weighted return for investment account K equals the time-weighted return for investment account L, both of which are denoted by i. Find i.

Solution: For the dollar-weighted rate of return on account K, i must satisfy

$$100(1+i) - X\left(1+\frac{i}{2}\right) + 2X\left(1+\frac{i}{4}\right) = 125 \quad \Rightarrow \quad X + 100i = 25.$$

On the other hand, for account L we require (see (5.4) in the book)

$$i = \frac{F_1}{A}\cdot\frac{F_2}{F_1+C_1}\cdots\frac{B}{F_k+C_k} - 1 = \frac{125}{100}\cdot\frac{105.8}{125-X} - 1 = \frac{5(105.8)}{500-4X} - 1 = \frac{529}{500-4(25-100i)} - 1,$$

or

$$i = \frac{529}{400+400i} - 1 \quad \Rightarrow \quad 400(1+i)^2 = 529 \quad \Rightarrow \quad i = \sqrt{\frac{529}{400}} - 1 = \frac{23}{20} - 1 = \frac{3}{20} = \boxed{15\%.}$$

9. An investor pays \$100,000 today for a 4-year investment that returns cash flows of \$60,000 at the end of each of years 3 and 4. The cash flows can be reinvested at 4.0% per annum effective. If the rate of interest at which the investment is to be valued is 5.0%, what is the NPV of this investment today?

A) -\$1398

B) -\$699

C) \$699

D) \$1398

E) \$2629

Solution: The only cash flow that can be reinvested is the \$60,000 that occurs at the end of year 3 and has value \$60,000(1.04) = \$62,400 at the end of the investment. The NPV at time $t = 0$ is thus

$$NPV = -100{,}000 + \frac{62{,}400}{1.05^4} + \frac{60{,}000}{1.05^4} \simeq \boxed{\$689.78.}$$

$\therefore$ **The correct answer is C.**