Math 312 - Mathematical Theory Interest - I - Exam 1 Practice Problems

1. Please circle either T (true) or F (false) for each of the below statements. You do NOT need to show your work.

I)   T   F   If the effective annual interest rate is 6% then $i^{(2)} = 3\%$.

II)   T   F   The future value of 1 after t years is ν^{-t}.

III)   T   F   If $i = 10\%$, $\ddot{s}_{\overline{10}|i} = 17.5312$.

IV)   T   F $(Ia)_{\overline{n}|i} + (Da)_{\overline{n}|i} = a_{\overline{n}|i}$.

V)   T   F $(Ia)_{\overline{\infty}|i} = \frac{1}{i^2}$.

VI)   T   F   If the annual continuous interest rate is 11%, then the nominal annual rate, compounded quarterly is approximately 11.1526%

VII)   T   F   If 1 is the initial value of an investment growing with force of interest $\delta(t) = t^2$, then the value of the investment at time $t = 10$ is e^{100}.

VIII)   T   F   If you invest \$100 at a continuous annual interest rate is $r > 0$, it $\ln(2)/r$ years to double the account to \$200.

IX)   T   F   One application of annuities is retirement planning.

X)   T   F   A perpetuity due refers to the valuation of a perpetuity one period before the first payment.

2. (10 total points) You invest 4200 into an account today.

   I) If in 15 years the account is worth 7500, find the nominal annual interest rate, convertible monthly.

   II) If, instead, the force of interest is $\delta(t) = 1/(t+2)$ and in X years your account is worth 16800, find X.

3. A payment of $X is made at time 0 that accumulates for 5 years at 7% annual effective interest. If, instead, a simple discount rate, d, is to be used, find d such that the payment of $X at time 0 has the same accumulated value at time 5.

   A) 1.40%

   B) 3.63%

   C) 5.19%

   D) 5.74%

   E) 6.54%

4. Consider the following:

- A one-time investment is made at time $t = 0$ into Fund A with an effective annual interest rate of 3%.

- A one-time investment is made at time $t = 0$ into Fund B with an effective annual interest rate of 2.5%.

At the end of 20 years the total amount in the two funds is $10000. At the end of 31 years, the amount in Fund A is twice the amount in Fund B.

Find the total amount in the two funds at the end of 10 years.

A) $5732

B) $6602

C) $7472

D) $7569

E) $8123

5. A special type of annuity is made where $1 is paid every three years and is paid at the beginning of each three-year period, starting at time 0. If there are n payments in this annuity, the present value can be expressed as:

A) $\dfrac{1 - \nu^{3n}}{i}$ B) $\dfrac{1 - \nu^{3n}}{d}$ C) $\dfrac{1 - \nu^{3(n+1)}}{1 - \nu}$ D) $\dfrac{1 - \nu^{3n}}{1 - \nu^3}$ E) $\dfrac{1 - \nu^{3(n+1)}}{1 - \nu^3}$

6. Darren makes 10 annual deposits of X each into a fund earning 5% effective annual interest. The deposits accumulate to an amount that is just sufficient to allow her to withdraw \$10000 annually for 15 years, with the first withdrawal one year after the last deposit.

   Find X.

7. An annuity pays \$50 in the beginning of the first month, \$100 in the beginning of the second month, \$150 in the beginning of the third month, continuing on in this fashion for 96 months. This annuity earns a nominal annual interest rate of 12% convertible monthly. Calculate the accumulated value of the annuity one month after the final payment.

A) \$320,000

B) \$331,000

C) \$339,000

D) \$347,000

E) \$353,000

8. Consider the two annuity payment options:

- Annuity 1: Receive payments of X at the end of each year for n years. The present value of the annuity is $493 \text{\euro}$.

- Annuity 2: Receive payments of $3X$ at the end of each year for $2n$ years. The present value of the annuity is $2748 \text{\euro}$.

Both present values are calculated at the same annual effective interest rate. Find v^n.

A) 0.86

B) 0.87

C) 0.88

D) 0.89

E) 0.90

9. Darius deposits $100 at the beginning of year 1 into an account that earns 3% annual effective interest. Each subsequent payment is 5% larger than the previous payment, and he makes deposits every six months for 10 years. Find the present value of these payments at time 0.

10. Alexa owns a special annuity that pays her $100 at the beginning of years 1 and 2, $200 at the beginning of years 3 and 4, $300 at the beginning of years 5 and 6, and so on until year 16. Calculate the accumulated value of these payments at the beginning of year 16 if the annuity pays an annual effective interest rate of 8

   A) $5,410
   B) $6,079
   C) $10,820
   D) $12,157
   E) $15,895