Math 245 - Quiz # 3 - Fall 2024 - Solution

1. (4 total points - 1 point each) Please circle either T (true) or F (false) for each of the below statements. There is no penalty for guessing. **Answers are in BOLD.**

   I) T **F** If $\lim_{(x,y)\to(0,0)} f(x,y) = 0$ for some function $f(x,y)$, then $f(x,y)$ is continuous at $(0,0)$.

   II) **T** F If $f(x,y) = \sin\left(\cos\left(e^{x^2y^4-x-y}\right)\right)$, then $f_{xy} = f_{yx}$.

   III) **T** F The domain $f(x,y) = \sqrt{x^2+y^2+x^4y^4}$ is $\mathbb{R}^2$.

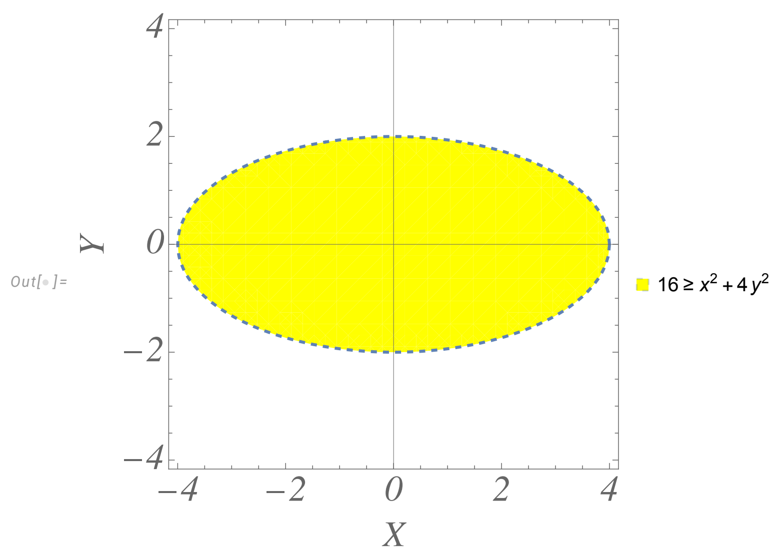
   IV) **T** F $\lim_{(x,y)\to(1,1)} x\sin(\pi y/2) = 1$.

2. (5 total points) Find and clearly graph the domain of

$$f(x,y) = \frac{1}{\sqrt{16-x^2-4y^2}}$$

in the x-y plane.

<u>*Solution:*</u> $1/\sqrt{x}$ is defined for $x > 0$. It follows that $f(x,y)$ is defined for $16-x^2-4y^2 > 0$ or $x^2+4y^2 < 16$, which is the inside of the ellipse $x^2+4y^2 = 16$. This is the yellow region depicted below, not including the ellipse itself.



3. (5 points) Evaluate

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y}{x^2-y}$$

if possible. If the limit does not exist, prove it.

<u>*Solution:*</u> The limit does not exist. For example, on the x-axis when $y=0$ and $x \to 0$ we have

$$f(x,y)|_{x\neq 0,\, y=0} = f(x,0) = \frac{x^2+0}{x^2-0} = \frac{x^2}{x^2} = 1.$$

On other hand, when $x=0$ and $y \to 0$ we have

$$f(x,y)|_{x=0,\, y\neq 0} = f(0,y) = \frac{0+y}{0-y} = \frac{y}{-y} = -1.$$

Therefore $f(x,y) \to 1$ as $(x,y) \to (0,0)$ along the x-axis while $f(x,y) \to -1$ as $(x,y) \to (0,0)$ along the y-axis.

4. (6 points) Let

$$f(x,z) = x \ln\left(z^2+x^2\right) + x\cos xz.$$

Find f_x and f_z.

<u>*Solution:*</u> Using the chain and product rule we have

$$\begin{aligned} f_x &= 1\cdot\ln\left(x^2+z^2\right) + x\cdot\frac{\left(x^2+z^2\right)_x}{x^2+z^2} + 1\cdot\cos xz - x\sin xz\cdot(xz)_x, \\ &= \boxed{\ln\left(x^2+z^2\right) + \frac{2x^2}{x^2+z^2} + \cos xz - xz\sin xz} \end{aligned}$$

and

$$\begin{aligned} f_z &= x\cdot\frac{\left(x^2+z^2\right)_z}{x^2+z^2} - x\sin xz\cdot(xz)_z, \\ &= \boxed{\frac{2xz}{x^2+z^2} - x^2\sin xz.} \end{aligned}$$