**Math 245 - Quiz # 1 - Fall 2024 - Solution**

**Directions:** Show **ALL** of your work. Answers that are not supported by calculations, graphs/diagrams, and explanations will **not** be given full credit.

1. (4 total points - 1 point each) Please circle either T (true) or F (false) for each of the below statements. There is no penalty for guessing. You DO NOT have to show your work to receive full credit. **Answers are in BOLD.**

   A) T **F** $\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}\right)/2$ is a unit vector.

   B) **T** F The radius of the sphere $x^2+y^2+z^2-2z=0$ is 1.

   C) **T** F The vector $\mathbf{u}=(1,2,3)$ is parallel to the vector $\mathbf{v}=(3,6,9)$.

   D) **T** F The set of points in $\mathbb{R}^3$ described by the equation $y-z=0$ is a plane parallel to the $x$-axis.

2. (6 points) Let $\mathbf{u}=3\hat{\mathbf{i}}-5\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ and $\mathbf{v}=-\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}$. Find $\mathbf{w}=2\mathbf{u}-5\mathbf{v}$.

   *Solution:* Using the vector algebra rules from class we have

$$\mathbf{w}=2\mathbf{u}-5\mathbf{v}=2\begin{pmatrix}3\\-5\\2\end{pmatrix}-5\begin{pmatrix}-1\\2\\-1\end{pmatrix}=\begin{pmatrix}2\cdot 3-5\cdot(-1)\\2\cdot(-5)-5\cdot 2\\2\cdot 2-5\cdot(-1)\end{pmatrix}=\begin{pmatrix}11\\-20\\9\end{pmatrix}=\boxed{11\hat{\mathbf{i}}-20\hat{\mathbf{j}}+9\hat{\mathbf{k}}.}$$

3. (5 points) Find the center and radius of the sphere

$$x^2+y^2+z^2-2y-4z-4=0.$$

   *Solution:* Completing the square yields

$$\begin{aligned}x^2+y^2+z^2-2y-4z-4 &= x^2+\overbrace{y^2-2y+1}^{(y-1)^2}-1+\overbrace{z^2-4z+4}^{(z-2)^2}-4-4,\\ &= x^2+(y-1)^2+(z-2)^2-9=0,\end{aligned}$$

   or $x^2+(y-1)^2+(z-2)^2=9=3^2$.

   $\therefore$ **we conclude that the center of the sphere is** $C=(0,1,2)$ **and the radius is** $R=3$**.**

4. (5 points) Find the unit vector $\hat{\mathbf{u}}$ that points from the point $P = (-1, 2, 3)$ to the point $Q = (3, -5, 7)$.

*Solution:* The position vector from $P$ *to* $Q$ is

$$\mathbf{r}_{PQ} = Q - P = (3, -5, 7) - (-1, 2, 3) = 4\,\hat{\mathbf{i}} - 7\,\hat{\mathbf{j}} + 4\,\hat{\mathbf{k}}.$$

It follows that the corresponding unit vector is

$$\begin{aligned} \hat{\mathbf{u}} &= \frac{\mathbf{r}_{PQ}}{|\mathbf{r}_{PQ}|} = \frac{4\,\hat{\mathbf{i}} - 7\,\hat{\mathbf{j}} + 4\,\hat{\mathbf{k}}}{\left|4\,\hat{\mathbf{i}} - 7\,\hat{\mathbf{j}} + 4\,\hat{\mathbf{k}}\right|} = \frac{4\,\hat{\mathbf{i}} - 7\,\hat{\mathbf{j}} + 4\,\hat{\mathbf{k}}}{\sqrt{4^2 + 7^2 + 4^2}} = \frac{1}{\sqrt{16 + 49 + 16}}\left(4\,\hat{\mathbf{i}} - 7\,\hat{\mathbf{j}} + 4\,\hat{\mathbf{k}}\right), \\ &= \boxed{\frac{1}{9}\left(4\,\hat{\mathbf{i}} - 7\,\hat{\mathbf{j}} + 4\,\hat{\mathbf{k}}\right).} \end{aligned}$$