Math 245 - Practice Quiz # 4

Directions: Show **ALL** of your work. Answers that are not supported by calculations, graphs/diagrams, and explanations will **not** be given full credit.

1. (4 total points - 1 point each) Please circle either T (true) or F (false) for each of the below statements. There is no penalty for guessing. You DO NOT have to show your work to receive full credit.

   I) T F The limits in polar coordinates for $\iint_\Omega f\,dA$ where $\Omega = \{(x,y)\ :\ 4 \le x^2+y^2 \le 9\}$ are $0 \le \theta \le 2\pi$ and $4 \le r \le 9$.

   II) T F If $f(x,y) = x^2 \sin\left(y^2\right)$, then

   $$\iint_{[-2,2]\times[0,\pi]} f(x,y)\,dA = \int_{-2}^{2} x^2\,dx \cdot \int_0^\pi \sin\left(y^2\right)\,dy.$$

   III) T F The differential area element in polar coordinates is $dA = dr\,d\theta$.

   IV) T F Given a smooth function $f(x,y)$ defined on $\mathbb{R}^2$, it is always true that

   $$\int_0^1 \int_0^x f(x,y)\,dy\,dx = \int_0^1 \int_y^1 f(x,y)\,dx\,dy.$$

2. Evaluate

$$\int_0^1 \int_0^{s^2} \cos\left(s^3\right)\,dt\,ds$$

Answer: $\sin(1)/3$.

3. Evaluate

$$\int_0^1 \int_x^1 e^{\frac{x}{y}}\, dy\, dx.$$

Answer: $(e-1)/2$.

4. Evaluate

$$\iint_\Omega 1 + y\, dA,$$

where Ω is the semicircular disk $\Omega := \big\{(x,y) \in \mathbb{R}^2 \ : \ x^2 + y^2 \leq 9 \quad \text{and} \quad y \geq 0\big\}$.

Answer: $9(\pi + 4)/2$.