Math 245 - Practice Problems for Quiz # 3

1. Please circle either T (true) or F (false) for each of the below statements. There is no penalty for guessing. **Answers are in BOLD.**

   I) T F The chain rule for smooth functions $u(x,y)$, $x(s,t)$, and $y(s,t)$ says that $z(s,t) = u(x(s,t), y(s,t))$ has the second partial derivative

   $$\frac{\partial^2 z}{\partial t^2} = u_{xx}x_t^2 + 2u_{xy}x_t y_t + u_{yy}y_t^2.$$

   II) T F $\lim_{(x,y)\to(2,1)}(2x - 3y) = 1$.

   III) T F The domain of $\ln(x^2 + y^2)$ is $\mathbb{R}^2$.

   IV) T F The chain rule for smooth functions $u(x,y)$, $x(t)$, and $y(t)$ says that $z(t) = u(x(t), y(t))$ has the derivative

   $$\frac{dz}{dt} = u_{xx}x'(t) + 2u_{xy}x'(t)y'(t) + u_{yy}y'(t).$$

   V) T F For any linear function $f(x,y)$, $f_x = f_y = 0$.

   VI) T F If $f(x,y)$ is continuous at $(a,b) \in \mathbb{R}^2$, then $\lim_{(x,y)\to(a,b)} f(x,y) = 0$.

2. Compute the below partial derivatives:

   $$\frac{\partial}{\partial x}\left(x^2 y - \ln(x+y)\right) \quad \text{and} \quad \left(\sin^2\left(\frac{x+y}{x-y}\right)\right)_y.$$

3. Find and graph in the x-y plane the domain of

   $$f(x,y) = \frac{xy}{1 - x^2 - y^2}.$$

4. Determine whether or not the limit

   $$\lim_{(x,y)\to(0,0)} \frac{x^2 y - x^2 - y^2}{x^2 + y^2}$$

   exists. If it does not, prove your conclusion. If it does, demonstrate why and find its value?

5. Compute $\partial/\partial x$ and $\partial/\partial y$ for both $f(x,y)$ and $g(x,y)$ below:

   $$f(x,y) = 3x^2y^5 - \ln\left(xy^2\right) \quad \text{and} \quad g(x,y) = \tan^{-1}\left(\frac{x}{xy+1}\right) + \tan\left(e^{x-y}\right).$$

6. Use the chain rule to compute u_s and u_t if

   $$u(x,y) = e^{x^2-y^2} \quad \text{with} \quad x(s,t) = \frac{s}{t+1} \quad \text{and} \quad y(s,t) = \sec(st).$$