Math 245 - Practice Quiz # 2 - Solution

Directions: Show **ALL** of your work. Answers that are not supported by calculations, graphs/diagrams, and explanations will **not** be given full credit.

1. (4 total points - 1 point each) Please circle either T (true) or F (false) for each of the below statements. There is no penalty for guessing. You DO NOT have to show your work to receive full credit.

   I) T **F** The equation $3x + 4y - 5z + xy = 5$ describes a plane in $\mathbb{R}^3$.

   II) T **F** A normal vector to the plane $2x - y + 2z = 0$ is the vector $(2,\ 1,\ 2)$.

   III) T **F** If $\mathbf{u} \in \mathbb{R}^3$, then $\mathbf{u} \times \mathbf{u} = \mathbf{u}$.

   IV) T **F** The line $x = t$, $y = -t$, $z = 1$ intersects the plane $x + y + z = 0$.

2. (3 points) If $\mathbf{u} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{v} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, find $\mathbf{u} \times \mathbf{v}$.

   Solution: Using the determinant method we have

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & -2 & 1 \\ 2 & 1 & 1 \end{vmatrix} = \begin{vmatrix} -2 & 1 \\ 1 & 1 \end{vmatrix} \hat{\mathbf{i}} - \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} \hat{\mathbf{j}} + \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} \hat{\mathbf{k}},$$

$$= (-2-1)\hat{\mathbf{i}} - (1-2)\hat{\mathbf{j}} + (1-(-4))\hat{\mathbf{k}} = \boxed{-3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 5\hat{\mathbf{k}}.}$$

3. (3 total points) Find the parametric and symmetric equations of the line passing through $(-1, 2, -5)$ and in the direction of $\mathbf{u} = (3, 3, 7)$.

   Solution: The vector equation is $\mathbf{r}(t) = (-1, 2, -5) + (3, 3, 7)t$ so that the parametric equation are

$$\boxed{x = -1 + 3t, \quad y = 2 + 3t, \quad \text{and} \quad z = -5 + 7t.}$$

   Solving for t in each equation above we have the symmetric equations:

$$t = \boxed{\frac{x+1}{3} = \frac{y-2}{3} = \frac{z+5}{7}.}$$

4. (4 points) Find the point of intersection between the line $x = -1 + t$, $y = 2 - 2t$, $z = 3 - 4t$ and the plane $3x - y + z = 5$.

<u>*Solution:*</u> Substituting the parametric line equations into the equation of the plane gives

$$3x - y + z = 5 = 3(-1+t) - (2-2t) + (3-4t) = -3 + 3t - 2 + 2t + 3 - 4t = t - 2.$$

Therefore, $t - 2 = 5$ or $t = 7$ so that the point of intersection is

$$x = -1 + 7 = 6, \quad y = 2 - 2(7) = -12, \quad \text{and} \quad z = 3 - 4(7) = -25$$

or $\boxed{(x, y, z) = (6, -12, -25).}$

5. (6 points) Find the equation of the plane passing through (1,1,1) and perpendicular to the line of intersection between $2x - y + z = 1$ and $x + y - z = 3$.

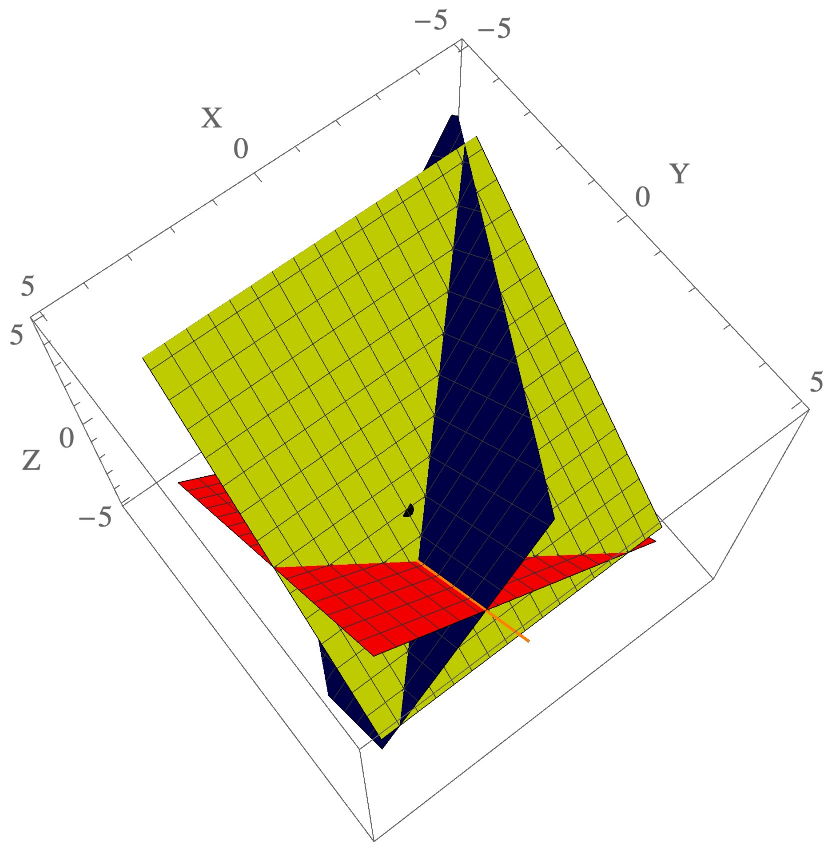
<u>*Solution:*</u> The respective plane normals are $\mathbf{n}_1 = (2, -1, 1)$ and $\mathbf{n}_2 = (1, 1, -1)$ so the line of intersection direction is

$$\mathbf{u} = \mathbf{n}_1 \times \mathbf{n}_2 = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = (1-1)\hat{\mathbf{i}} - (-2-1)\hat{\mathbf{j}} + (2+1)\hat{\mathbf{k}} = (0, 3, 3).$$

The plane <u>perpendicular</u> to the intersection line will have a normal parallel to $\mathbf{u} = 3\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$. For example, $\hat{\mathbf{j}} + \hat{\mathbf{k}}$ will work. It follows that the desired plane has an equation of the form $y + z = d$. Requiring that the plane passes through $x = y = z = 1$ forces d to satisfy $y + z = 1 + 1 = d = 2$. Therefore, the final equation of the plane is

$$\boxed{y + z = 2.}$$

A graph of the two planes (red & blue), the line of intersection (orange), the perpendicular plane $y + z = 2$ (yellow), and the point (1,1,1) (black) is on the next page.



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−5
X
0
Y
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5
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