Math 245 - Practice Quiz # 1 - Solution

Directions: Show **ALL** of your work. Answers that are not supported by calculations, graphs/diagrams, and explanations will **not** be given full credit.

1. (4 total points - 1 point each) Please circle either T (true) or F (false) for each of the below statements. There is no penalty for guessing. You DO NOT have to show your work to receive full credit. **Answers are in BOLD.**

   A) **T** F The angle between $\mathbf{w}$ and $-\mathbf{w}$ is π.

   B) T **F** The set of all points a distance 1 from the z-axis in $\mathbb{R}^3$ is a sphere.

   C) **T** F $\mathbf{u} = \hat{\mathbf{i}} - \hat{\mathbf{k}}$ is perpendicular to $\mathbf{v} = 3\hat{\mathbf{j}}$.

   D) T **F** There exists $\mathbf{u} \in \mathbb{R}^3$ such that $\mathbf{u} \cdot \mathbf{0} \neq 0$.

2. (16 total points) Let $\mathbf{u} = -2\hat{\mathbf{i}} + 3\hat{\mathbf{k}}$ and $\mathbf{v} = 4\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$.

   A) (4 points) Find $2\mathbf{u} + \mathbf{v}$.

   Solution:

$$2\mathbf{u} + \mathbf{v} = 2(-2, 0, 3) + (4, -2, -4) = (-4, 0, 6) + (4, -2, -4) = (0, -2, 2) = \boxed{-2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}.}$$

   B) (6 points) Calculate $|\mathbf{v}|$. Use your answer to find the unit vector $\hat{\mathbf{v}}$ in the direction $\mathbf{v}$.

   Solution:

$$|\mathbf{v}| = \sqrt{4^2 + (-2)^2 + (-4)^2} = \sqrt{16 + 4 + 16} = \sqrt{36} = \boxed{6.}$$

   The unit vector $\hat{\mathbf{v}}$ is therefore

$$\hat{\mathbf{v}} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{1}{6}(4, -2, -4) = \left(\frac{4}{6}, -\frac{2}{6}, -\frac{4}{6}\right) = \boxed{\frac{2}{3}\hat{\mathbf{i}} - \frac{1}{3}\hat{\mathbf{j}} - \frac{2}{3}\hat{\mathbf{k}}.}$$

   C) (6 points) Use your answer to part (B) to find the projection of the vector $\mathbf{u}$ *onto* the vector $\mathbf{v}$.

   Solution: The projection of $\mathbf{u}$ onto $\mathbf{v}$, denoted by $P_{\mathbf{v}}(\mathbf{u})$, is given by the formula

$$\begin{aligned} P_{\mathbf{v}}(\mathbf{u}) &= (\mathbf{u} \cdot \hat{\mathbf{v}})\hat{\mathbf{v}} = \frac{(\mathbf{u} \cdot \mathbf{v})}{|\mathbf{v}|^2}\mathbf{v} = \frac{(-2, 0, 3) \cdot (4, -2, -4)}{6^2}(4, -2, -4), \\ &= \frac{-8 + 0 - 12}{36}(4, -2, -4) = -\frac{5}{9}(4, -2, -4) = \boxed{\frac{1}{9}(-20,\ 10,\ 20).} \end{aligned}$$