Math 245 - Project #4 - Solving Optimization Problems with *Mathematica*

DUE: 11:59PM on Monday, December 02, 2024

Objective: To use *Mathematica* to

- plot surfaces and use its numerical functions to identify extrema of complicated functions;
- solve constrained optimization problems that result in systems many equations and unknowns

Questions: For all of the below problems, in addition to using Mathematical to generate images, you must provide appropriate analytical support for your solutions.

1. A manufacturer of lawn furniture makes two types of lawn chairs, one with a wood frame and one with a tubular aluminum frame. The wood-frame model costs \$18 per unit to manufacture, and the aluminum-frame model costs \$10 per unit. The company operates in a market where the number of units that can be sold depends on the price. It is estimated that in order to sell x units per day of the wood-frame model and y units per day of the aluminum-frame model, the selling price should be $10+31x^{-0.5}+1.3y^{-0.2}$ \$/unit for the wood-frame chairs, and $5+15y^{-0.4}+0.8x^{-0.08}$ \$/unit for the aluminum-frame chairs. The goal of this problem is to find the production levels (x, y) so that profit is maximized.

   Note: This problem will require your group to explore and use the documentation that comes with the *Mathematica* software to use the functions NSolve, Maximize, and Minimize. You should ask me questions as they come up and I will help you out.

   A) Derive the profit function $p(x, y)$ for this problem. What constraints should be imposed on x and y, if any?

   B) Graph $p(x, y)$ over a relevant domain of $\mathbb{R}^2$. How many maxima do you see? Approximate the location of any you see before moving on in this problem.

   C) Find the two nonlinear equations for x and y that must be satisfied by any critical point by evaluating ∇f.

   D) Try to use the command "NSolve" command to find solutions to *system* of two nonlinear equations in x and y that result from stating the vector equation $\nabla f(x, y) = (0, 0)$. What happens when you use the command? If you find any solutions, asses whether or not they are consistent with the graph from part (B). Also, use the second-derivative test to asses the character of any critical points that you find.

E) Use the “Minimize” command to find solutions to the problem

$$\min_{x,\, y>0} |\nabla f|^2.$$

Are the solutions you find consistent with the graph from part (B)? Explain why solutions to this minimization problem should be related to solutions to the original maximum profit problem.

F) Use the “Maximize” command to find solutions to the original maximum profit problem. Are the solutions you find consistent with the graph from part (B)? What about part (E)?

2. The following is a problem that shows up in the study of chemical equilibrium. Let

$$f(x, y) = a - x - y \quad \text{and} \quad g(x, y) = (a - x - y)^2 - bxy^3.$$

Use Lagrange multipliers to find the values of $(x, y) \in \mathbb{R}^2$ that minimize $f(x, y)$ subject to the constraint that $g(x, y) = 0$ and $x,\ y \le 0$ when $a = -1/10$ and $b = 1$.