Math 245 - Project # 2 - Exploring Curves with *Mathematica*

DUE: 11:59PM on Monday, October 14, 2024

Objective: To use *Mathematica* to plot curves in 3D and to numerically compute arc length and construct tangent/normal/binormal coordinate systems near at a curve.

Questions: For all of the below problems, in addition to using Mathematical to generate images and compute arc lengths, you must provide full analytical support for your solutions.

1) Graph the curve

$$\mathbf{r}(t) = \frac{1}{2}\sin(2t)\,\hat{\mathbf{i}} + \frac{1}{2}(1-\cos(2t))\,\hat{\mathbf{j}} + \cos t\,\hat{\mathbf{k}}$$

for $0 \le t \le 2\pi$. *Prove analytically* that the curve must lie on a sphere centered at the origin. Can you think of a new curve that lies on a sphere, also centered at the origin, but with radius equal to the number of the month in which you were born? Given the equation of this new curve and graph it using *Mathematica*'s *ParametricPlot3D* command

2) Graph the elliptical helix

$$\mathbf{r}(t) = 2t\,\hat{\mathbf{i}} + 3\cos\,t\,\hat{\mathbf{j}} + 2\sin t\,\hat{\mathbf{k}}$$

for $0 \le t \le 2\pi$ using the *ParametricPlot3D* command. Use the *NIntegrate* function in *Mathematica* to find the approximate length of the curve. What answer do you get if you just use the "Integrate" command? How is this output related to the NIntegrate result? Explain.

3) Consider the curve

$$\mathbf{r}(t) = \cos(3t)(3+\cos(4t))\,\hat{\mathbf{i}} + \sin(3t)(3+\cos(4t))\,\hat{\mathbf{j}} + \sin(4t)\,\hat{\mathbf{k}}.$$

A) Plot $\mathbf{r}(t)$ for $0 \le t \le 2\pi$. Describe what you see on the screen. What do you find interesting about this curve?

B) Construct the tangent, normal, and binormal lines to the curve at the point $(\sqrt{2},\ \sqrt{2},\ 0)$. Show in a single plot, distinct from your plot from Part A, all three lines *plus* the curve near the point of tangency. That is, you don't need to show in this plot the entire curve you found in Part A.