Math 245 - Exam # 3 Practice Problems - Fall 2019

1. Consider the region Ω in the xy plane that is bounded by the curves $x = 5 - y^2$ and $x = y^2 - 13$.

   (a) Carefully sketch the region Ω in the xy plane.

   (b) Calculate the area of the region Ω. *Answer:* 72.

2. Evaluate the integral by reversing the order of integration:

$$\int_0^1 \int_{3y}^3 e^{x^2}\, dx\, dy.$$

   Answer: $\left(e^9 - 1\right)/6$.

3. Evaluate the iterated integrals:

   (a)

$$\int_0^1 \int_1^2 xy\, dx\, dy$$

   Answer: $3/4$.

   (b)

$$\int_0^1 \int_0^{s^2} \cos\left(s^3\right)\, dt\, ds$$

   Answer: $\sin(1)/3$.

4. Find the area of the region bounded by the spiral $r = 2\theta$ for $0 \le \theta \le \pi$, and the x-axis. *Answer:* $2\pi^3/3$.

5. Consider the iterated integral

$$I = \int_0^1 \int_x^1 e^{\frac{x}{y}}\, dy\, dx.$$

   (a) Use the limits in the above integral to draw the corresponding region of integration for I in the x-y plane.

   (b) Interchange the limits of integration to evaluate I. *Answer:* $(e - 1)/2$.

6. The semicircular disk

$$\Omega := \left\{(x, y) \in \mathbb{R}^2 \ : \ x^2 + y^2 \leq 9 \quad \text{and} \quad y \geq 0\right\}$$

has a mass-density of $\rho(x, y) = 1 + y$ kg/m^2. Find the mass M and the center of mass $(\overline{x}, \overline{y})$ of Ω. *Answer:* $M = 9(\pi + 4)/2$, $(\overline{x}, \overline{y}) = (0, 2(18 + 81\pi/8)/(9 + 4\pi))$.

7. Find the volume of the solid bounded by the paraboloids $z = 2x^2 + y^2$ and $z = 27 - x^2 - 2y^2$. *Answer:* $243\pi/2$.

8. Rewrite the order of integration for

$$I = \int_0^1 \int_{-2}^2 \int_0^{\sqrt{4-y^2}} dz\, dy\, dx$$

in the order $dy\, dz\, dx$ and then evaluate I. *Answer:* 2π.

9. Consider the integral

$$I := \int_{-2}^2 \left[\int_0^{\sqrt{4-y^2}} \left(\int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2 \sqrt{x^2 + y^2 + z^2}\, dz \right) dx \right] dy.$$

(a) Carefully sketch the region of integration indicated by the limits in I.

(b) Evaluate I using spherical coordinates. *Answer:* $64\pi/9$

10. Find the average of the squared distance between the origin and points in the cylinder bounded by $x^2 + y^2 = 4$ and the planes $z = 0$ and $z = 2$. *Answer:* $10/3$.

11. Let $\mathcal{B}$ be the solid bounded by the parabolic cylinders $y = x^2$ and $x = y^2$, and the planes $z = 0$ and $z = x + y$. Evaluate

$$\iiint_{\mathcal{B}} xy\, dV.$$

Answer: $3/28$.